SE 422
Advanced Photogrammetry

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REMINDER:

- The midterm exam will be held on:

  Tuesday 10/1/2023, during the Lab time (1:30pm – 3:00pm)

<u>(Tentative topics of the exam until the end of 8-Parameter Transformation)</u>

Two-Dimensional (2D) Coordinate Transformation

Two-Dimensional (2D) Coordinate Transformation

- Depending on the required accuracy or the available common points, **either** one of the following 2D transformations is carried out:

(1) Similarity Transformation (4 Parameters Transf. or Helmert Transf.)
- 4 unknown parameters: Linear and nonlinear

(2) Affine Transformation
- 6 unknown parameters: Linear and nonlinear

(3) Bilinear Transformation
- 8 unknown parameters: Linear and nonlinear

(4) Projective Transformation
- 8 unknown parameters: Linear and nonlinear

2D Conformal Coordinate Transf.

- Process for converting from one coordinate system to another is known as *coordinate transformation.*
- Two-dimensional → plane surfaces.
- Conformal → True Shape is preserved after transformation
- Coordinates of Two points must be known in both coordinate systems (arbitrary and final).
- Accuracy is improved by choosing these two points as far apart as possible
- If there are more than two control points, an improved solution may be obtained using the LS technique

Three Basic Steps of 2D Transformation

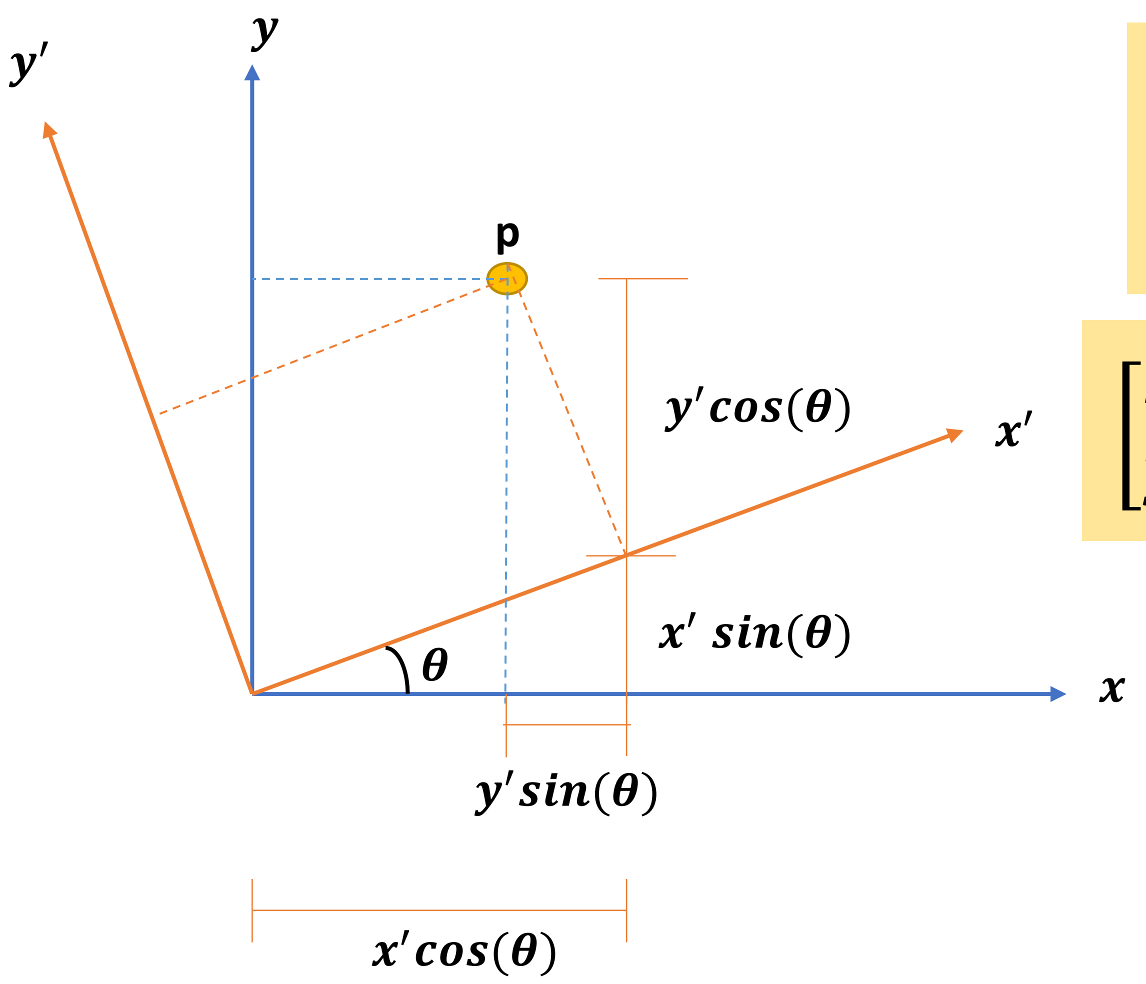
- 1) **Scale Change**: $s = \frac{ab_{dis.\,on\,image}}{AB_{dis.on\,ground}}$
- 2) **Rotation**: From a tangential angle
- 3) **Shifts**: use the original model to get the shifts in both directions
- If the scale ($\boldsymbol{\lambda}$) is eliminated → Rigid body transformation.

Applications of 2D Conformal Coordinate Transf.

- **Rectifying the image measurements**
- **Mosaics**: Continuous pictures of the terrain
- **Initial approximation value:** of angle κ^o for space resection (SR)

2D Similarity Transformation (4-Parameter Transformation)

Two-Dimensional (2D) Similarity Transf.



$$x = x' \cos(\theta) - y' \sin(\theta)$$

$$y = x' \sin(\theta) + y' \cos(\theta)$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

Two more things are left ?

Rotation Matrix in 2D

- **M** is the rotation matrix corresponding to rotation angle θ. This is an orthogonal matrix having orthonormal column and row vectors and it has the properties:
- $\boldsymbol{M^{-1} = M^{T}}$ **and** $\boldsymbol{M^{T}M = I}$

$$\mathbf{M}= \begin{bmatrix} \boldsymbol{cos(\theta)} & \boldsymbol{-sin(\theta)} \\ \boldsymbol{sin(\theta)} & \boldsymbol{cos(\theta)} \end{bmatrix} \Rightarrow M^{-1} = M^{T} = \begin{bmatrix} \boldsymbol{cos(\theta)} & \boldsymbol{sin(\theta)} \\ \boldsymbol{-sin(\theta)} & \boldsymbol{cos(\theta)} \end{bmatrix}$$

Two-Dimensional (2D) Similarity Transf.

$$x = x'\lambda\cos(\theta) - y'\lambda\sin(\theta) + t_x$$

$$y = x'\lambda\sin(\theta) + y'\lambda\cos(\theta) + t_y$$

Nonlinear Model

Why?

$$\begin{bmatrix} x \\ y \end{bmatrix} = \lambda \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Nonlinear Model in matrix form

$$x = \lambda M x' + T$$

Nonlinear Model in compact form

Two-Dimensional (2D) Similarity Transf.

If $\boldsymbol{X}$ and $\boldsymbol{Y}$ are constants (i.e., the σ_i is very small or the weight is very large).

Let: $a = \lambda \cos\theta$, $b = \lambda \sin\theta$, $c = tx$, and $d = ty$

$$\boldsymbol{x_i = a\,x'_i - b\,y'_i + c}$$

$$\boldsymbol{y_i = bx'_i + a\,y'_i + d}$$

Linear Model

The linear model of 2D similarity transformation can be solved. Then the parameters can be used as approximation values for the nonlinear model.

$$\begin{bmatrix} \boldsymbol{x_i} \\ \boldsymbol{y_i} \end{bmatrix}_{\boldsymbol{2nx1}} = \begin{bmatrix} \boldsymbol{x'_i} & \boldsymbol{-y'_i} & \boldsymbol{1} & \boldsymbol{0} \\ \boldsymbol{y'_i} & \boldsymbol{x'_i} & \boldsymbol{0} & \boldsymbol{1} \end{bmatrix}_{\boldsymbol{2nX4}} \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \\ \boldsymbol{c} \\ \boldsymbol{d} \end{bmatrix}_{\boldsymbol{4X1}}$$

Two-Dimensional (2D) Similarity Transf.

- Extracting the physical parameters:

$$\boldsymbol{a = \lambda\, cos(\theta)}, \qquad \boldsymbol{b = \lambda\, sin(\theta)},$$

$\boldsymbol{\lambda}$	$\boldsymbol{\theta}$	$\boldsymbol{t_x}$	$\boldsymbol{t_y}$
$a^2 + b^2 = \lambda^2 \cos^2(\theta) + \lambda^2\, sin^2(\theta)$ $a^2 + b^2 = \lambda^2(\cos^2(\theta) + sin^2(\theta))$ $a^2 + b^2 = \lambda^2$ $\boldsymbol{\lambda} = \sqrt{a^2 + b^2}$	$\frac{b}{a} = \frac{\lambda \sin(\theta)}{\lambda \cos(\theta)}$ $\theta = tan^{-1}(b/a)$	$t_x = c$	$t_y = d$

Direction of all Quadrants

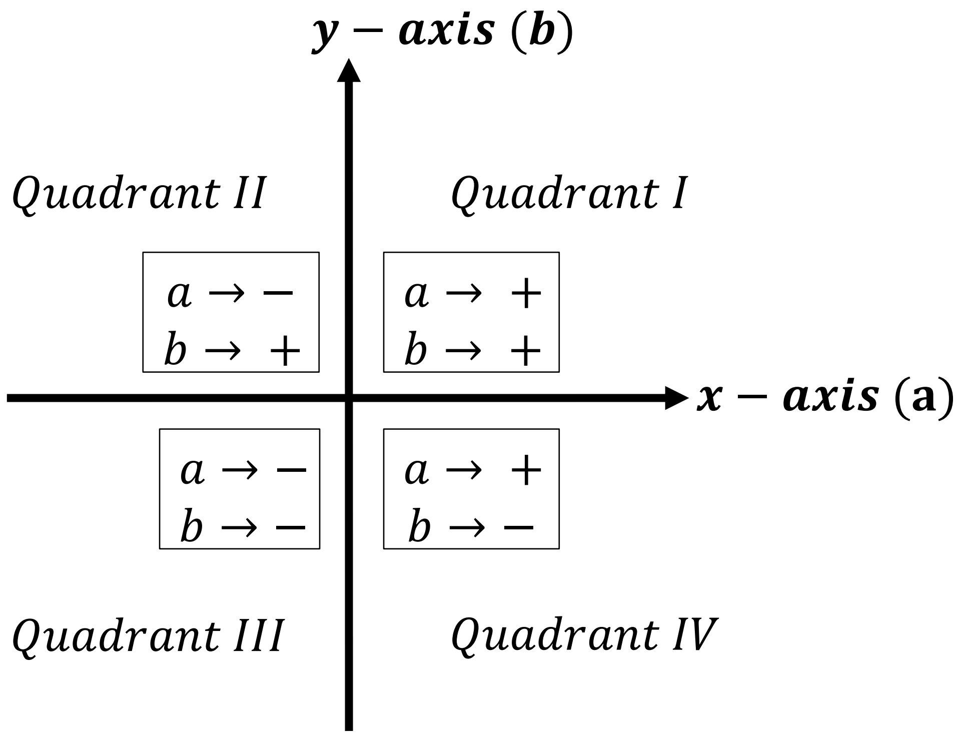


Figure: Four Quadrants

To get θ in correct quadrant using MATLAB → $\boldsymbol{\theta = atan2(b, a)}$

Be careful, when using the calculator, you must return $\boldsymbol{\theta}$ to its correct quad.

Example 1: Linear Model of 2D Transf.

- Given two points of the fiducial marks, together with observations of these two points. Compute the coordinates of point number 3 using the four-parameter (2D similarity) transformation.

No.	x'_i	y'_i	x_i	y_i
1	632.17	121.45	1100.64	1431.09
2	355.2	-642.07	1678.39	254.15
3	1304.81	596.37	---	---

Solution by MATLAB

- From *Math 107 (Linear Algebra)*, the solution for the linear system ($\boldsymbol{AX} = \boldsymbol{b}$) is:
- $\boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{b}$ (Assuming that A is square matrix)
- Remember from ***SE 331*** course, If $\boldsymbol{A}$ is not square matrix, the solution is $\boldsymbol{X} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{b}$.
- **The main objective to solve for the unknown parameters ($\boldsymbol{\theta}, \boldsymbol{\lambda}, \boldsymbol{t_x}, \boldsymbol{t_y}$).**
- **The model is nonlinear. As a result, the following steps are needed:**
  - Initial approximation values
  - Partial derivatives of the model
  - Iterative process solution using MATLAB (such as Newton Iteration)

No.	x'_i	y'_i	x_i	y_i
1	632.17	121.45	1100.64	1431.09
2	355.2	-642.07	1678.39	254.15
3	1304.81	596.37	---	---

Solution

$$\begin{bmatrix} x_i \\ y_i \end{bmatrix}_{2nx1} = \begin{bmatrix} x'_i & -y'_i & 1 & 0 \\ y'_i & x'_i & 0 & 1 \end{bmatrix}_{2nX4} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}_{4X1}$$

In equation form: ➔ $632.17\, a + 121.45\, b + c + 0 * d = 1100.64$

$121.45\, a - 632.17\, b + 0 * c + d = 1431.09$

$355.20\, a + (-642.07)\, b + c = 1678.39$

$(-642.07)\, a - 355.20\, b + d = 254.15$

In matrix form: ➔ $A = \begin{bmatrix} 632.17 & -121.45 & 1 & 0 \\ 121.45 & 632.17 & 0 & 1 \\ 355.20 & 642.07 & 1 & 0 \\ -642.07 & 355.20 & 0 & 1 \end{bmatrix}; L = \begin{bmatrix} 1100.64 \\ 1431.09 \\ 1678.39 \\ 254.15 \end{bmatrix}; X = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

This example can be solved by either (Ax=L ➔ $X = A^{-1}L$) or ($X = (A^T W A)^{-1} A^T W L$)

$$X = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1.1196 \\ 1.1628 \\ 534.0657 \\ 559.9934 \end{bmatrix}$$

$$\begin{bmatrix} x_3 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1301.528 \\ 2745.01 \end{bmatrix}$$

Now, extract the physical parameters: $\lambda = \sqrt{a^2 + b^2}$ =1.6142

$\theta = tan^{-1}(b/a) = 0.8043\ rad = 46.0844^0$

$t_x = 534.0657\ mm$

$t_y = 559.9934\ mm$

Example 2: Linear Model of 2D Transf.

- Given 7 points of the fiducial marks, together with observations of these 7 points. Compute the coordinates of 7 and 8 using the four-parameters similarity transformation.

No.	x_i'	y_i'	x_i	y_i
1	98.28	200.32	101	201
2	101.22	199.8	104	201
3	98.99	204.21	101	205
4	99.75	203.06	102	204
5	102.54	201.57	105	203
6	102.91	203.54	105	205
7	95.01	206	---	---
8	99.00	210	---	---

** note that here A matrix will not be a square matrix (we cannot use $\boldsymbol{X = A^{-1}b}$ in this case)

Constructing the Design matrix ($\boldsymbol{A}$ or $\boldsymbol{J}$) and misclosure vector ($\boldsymbol{L}$ or $\boldsymbol{K}$ or $\boldsymbol{f_i}$)

- **for** $i = 1{:}n$

$$\boldsymbol{A} = \begin{bmatrix} x_1' & -y_1' & 1 & 0 \\ y_1' & x_1' & 0 & 1 \\ x_2' & -y_2' & 1 & 0 \\ y_2' & x_2' & 0 & 1 \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ x_n' & -y_n' & 1 & 0 \\ y_n' & x_n' & 0 & 1 \end{bmatrix}; \boldsymbol{L} = \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ . \\ . \\ . \\ x_n \\ y_n \end{bmatrix}$$

→ in MATLAB, $\boldsymbol{A} = \begin{bmatrix} \boldsymbol{x'(i)} & \boldsymbol{-y'(i)} & \boldsymbol{1} & \boldsymbol{0} \\ \boldsymbol{y'(i)} & \boldsymbol{x'(i)} & \boldsymbol{0} & \boldsymbol{1} \end{bmatrix}; \boldsymbol{L} = \begin{bmatrix} \boldsymbol{x(i)} \\ \boldsymbol{y(i)} \end{bmatrix}$

end

$$\boldsymbol{X = (A^T W A)^{-1} A^T W L},$$

Partial Derivatives of the 2D Tranf.

$$J_{2n\,\mathrm{x}\,4} = \begin{bmatrix} \frac{\partial F_1}{\partial \lambda}, \frac{\partial F_1}{\partial \theta}, \frac{\partial F_1}{\partial t_x}, \frac{\partial F_1}{\partial t_y} \\ \frac{\partial F_2}{\partial \lambda}, \frac{\partial F_2}{\partial \theta}, \frac{\partial F_2}{\partial t_x}, \frac{\partial F_2}{\partial t_y} \end{bmatrix}_{2n\mathrm{x}4}$$

(we will derive that in the lab)

Where

$$F_1 = x_i - \lambda x_i\, cos(\theta) + \lambda x_i' sin(\theta) - t_x$$

$$F_2 = y_i - \lambda x_i\, sin(\theta) - \lambda y_i'\, cos(\theta) - t_y$$

$$K = -F = -\begin{bmatrix} x_i - \lambda x_i\, cos(\theta) + \lambda x_i' sin(\theta) - t_x \\ y_i - \lambda x_i\, sin(\theta) - \lambda y_i'\, cos(\theta) - t_y \end{bmatrix}_{2n\,\mathrm{x}\,1}$$

Example: Nonlinear Model of 2D Transf.

We will do this example in the Lab (this week)

HW-2 will be available this week, and it is due next week.

While $(\boldsymbol{\Delta} < \mathbf{0.00000001})$

for $i = 1{:}n$

$$B = \begin{bmatrix} \frac{\partial F_1}{\partial \lambda}, \frac{\partial F_1}{\partial \theta}, \frac{\partial F_1}{\partial t_x}, \frac{\partial F_1}{\partial t_y} \\ \frac{\partial F_2}{\partial \lambda}, \frac{\partial F_2}{\partial \theta}, \frac{\partial F_2}{\partial t_x}, \frac{\partial F_2}{\partial t_y} \end{bmatrix}_{2nx4} ;$$

➔ (this will be derived in the lab.)

$$K = -\begin{bmatrix} x - \lambda x_i' \, cos(\theta) + \lambda y_i' \, sin(\theta) - \; t_x \\ y - \lambda x_i' \, sin(\theta) - \lambda y_i' \, cos(\theta) - \; t_y \end{bmatrix}_{2n\,x1}$$

end

*** Remember from SE331:** $\boldsymbol{\Delta} = \left(\mathbf{B^T W B}\right)^{-1} * \left(\mathbf{B^T W f}\right)$

$$\boldsymbol{\Delta}_{4x1} = \begin{bmatrix} \boldsymbol{\delta\lambda} \\ \boldsymbol{\delta\theta} \\ \boldsymbol{\delta t_x} \\ \boldsymbol{\delta t_y} \end{bmatrix} = (\boldsymbol{B^T W B})^{-1}(\boldsymbol{B^T W K})$$

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{\lambda} \\ \boldsymbol{\theta} \\ \boldsymbol{t_x} \\ \boldsymbol{t_y} \end{bmatrix}^{\boldsymbol{0ld}} + \begin{bmatrix} \boldsymbol{\delta\lambda} \\ \boldsymbol{\delta\theta} \\ \boldsymbol{\delta t_x} \\ \boldsymbol{\delta t_y} \end{bmatrix}$$

end